

Simplicity of the Holding Period Return

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Abstract

When asked, students of investments and finance often conjecture that the holding period return to a bond is perhaps its interest rate and to a common stock is often its growth rate or the rate of return achieved by the firm. Instead, the holding period return invariably is the yield to maturity applied to the bond and the discount rate applied to the common stock.

Keywords: Stocks, bonds, holding period return, discount rate, yield to maturity

A holding period return R is a ratio of future proceeds divided by its initial investment. For a bond it is $R = (B_1 - B_0 + iF)/B_0$ with bond valuations B_t at time t with interest payments of iF at interest rate i on face value F , and with a maturity M discounted at rate k which for bonds is called the yield to maturity. Interest payments on corporate bonds are often paid twice a year as half the annual amount of iF . The interest payments are an annuity as $iF/k(1 - 1/[1+k]^M)$ and the face value is $F/(1+k)^M$ or:

$$B_0 = iF/k(1 - 1/[1+k]^M) + F/(1+k)^M \text{ and}$$

$$B_1 = iF/k(1 - 1/[1+k]^{M-1}) + F/(1+k)^{M-1}.$$

Thus:

$$R = \{iF/k(1 - 1/[1+k]^{M-1}) + F/(1+k)^{M-1} - iF/k(1 - 1/[1+k]^M) - F/(1+k)^M + iF\} / \{iF/k(1 - 1/[1+k]^M) + F/(1+k)^M\},$$

canceling F :

$$R = \{i/k(1 - 1/[1+k]^{M-1}) + 1/(1+k)^{M-1} - i/k(1 - 1/[1+k]^M) - 1/(1+k)^M + i\} / \{i/k(1 - 1/[1+k]^M) + 1/(1+k)^M\},$$

expanding the annuities:

$$R = \{i/k - [i/k]/[1+k]^{M-1} + 1/(1+k)^{M-1} - i/k + [i/k]/[1+k]^M - 1/(1+k)^M + i\} / \{i/k - [i/k]/[1+k]^M + 1/(1+k)^M\},$$

canceling like terms:

$$R = \{-[i/k]/[1+k]^{M-1} + 1/(1+k)^{M-1} + [i/k]/[1+k]^M - 1/(1+k)^M + i\} / \{i/k - [i/k]/[1+k]^M + 1/(1+k)^M\},$$

multiplying by $(1+k)^M$:

$$R = [-i(1+k)/k + (1+k) + i/k - 1 + i(1+k)^M] / [i(1+k)^M/k - i/k + 1],$$

and multiplying by k :

$$R = [-i(1+k) + (1+k)k + i - k + ik(1+k)^M] / [i(1+k)^{M-i} + k],$$

and expanding:

$$R = [-i - ik + k + ik + i - k + ik(1+k)^M] / [i(1+k)^{M-i} + k],$$

and canceling:

$$R = [-ik + k + ik(1+k)^M] / [i(1+k)^{M-i} + k].$$

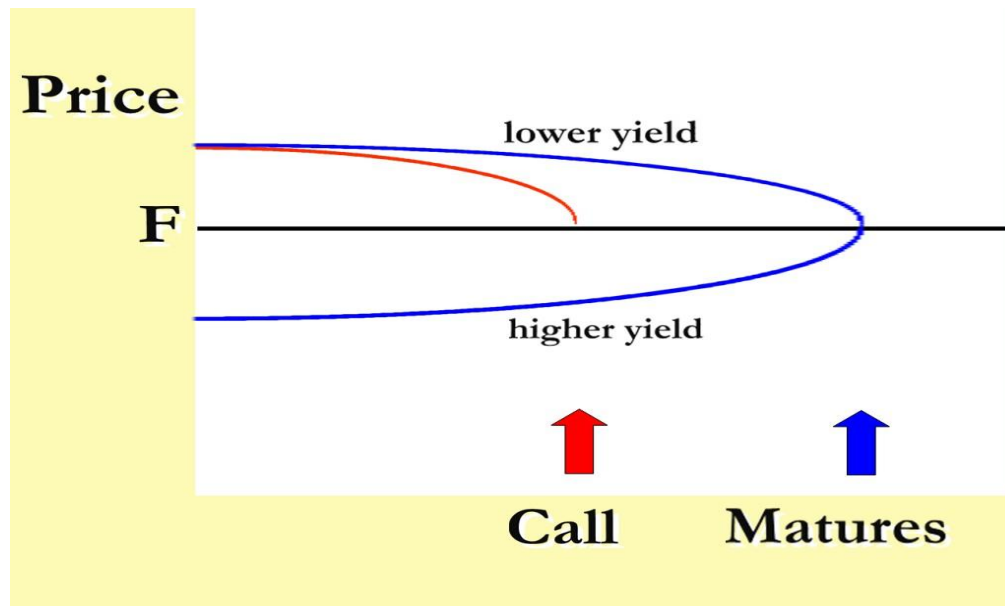
The numerator is a multiple of the denominator by k , therefore $R = k$.

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Here are some different examples of holding period returns for various bonds discounted at a 10 percent yield: Coupon

Interest	Maturity	Price	Maturity	Price	Yield
0	2	86.64	1	90.91	10
1	3	77.62	2	84.38	10
5	30	52.87	29	53.15	10
10	20	100.00	19	100.00	10
12	5	107.58	4	106.34	10
15	20	142.56	19	141.82	10

Do be wary of bonds trading at a premium and/or convertible bonds. For bonds trading at a premium, yield to call price and call date would be more appropriate.



For a common stock the holding period return R with valuations P_t and D_t at time t , the yearly return would be $R = (P_1 - P_0 + D_1)/P_0$. Often dividends are paid four times a year with an associated decrease in value on its ex-dividend date; exchange traded funds often pay monthly. A common stock with a dividend payment of D_t and discounted at rate k with growth rate g , is valued as:

$$P_0 = D_1 / (k - g) = D_0(1 + g) / (k - g);$$

$$\text{Substituting: } R = (D_1[1 + g] / [k - g] - D_1 / [k - g] + D_1) / (D_1 / [k - g]),$$

Canceling D_1 :

$$R = ([1 + g] / [k - g] - 1 / [k - g] + 1) / (1 / [k - g]),$$

Multiplying by $(k - g)$:

$$R = (1 + g - 1 + k - g)$$

Which results in $R = k$.

Here are some examples of different common stocks with different dividends and different growth rates discounted at a 10 percent discount rate:

Growth Rate	Current Dividend	Current Price	Next Dividend	Next Price	Yield
0	2.00	20.00	2.00	20.00	10
4	1.00	17.33	1.04	18.03	10
6	4.00	106.00	4.24	112.36	10
8	1.00	54.00	1.08	58.32	10

Do be wary that tax considerations were neutral here whereas in reality capital gains tax rates and the tax rates upon dividends may differ.

Note that $D_1 = E_1(1-b)$ where E is a firm's earnings, b is the firm's retention rate, and a firm's endogenous growth rate g may be determined by $g = br$ where r is the firm's rate of return. Earnings are achieved on the firm's assets A or $E_1 = A_0r$.

Thus:

$$P_0 = D_1/(k-g) = E_1(1-b)/(k-g) = A_0r(1-b)/(k-br).$$

Likewise $k = D_1/P+g = E_1(1-b)/P+br = A_0r(1-b)/P+br$. Where r is greater (less) than k , P will be valued greater (less) than A . However, when r is greater than k , a lesser dividend and a greater retention rate resulting in increased growth will increase valuations, whereas when r is less than k then a greater dividend and lesser retention and lower growth rates will increase valuations albeit still below asset valuation A . Consider A_0 equaling 100, with r equaling .12 and E_1 equaling 12, the valuations are:

b	1-b	g	D_1	$k=.14$	$k=.12$	$k=.10$
0	1	.00	12	85.7	100.0	120.0
1/4	3/4	.03	9	81.8	100.0	128.6
1/2	1/2	.06	6	75.0	100.0	150.0
3/4	1/4	.09	3	60.0	100.0	300.0

When r equals k , P equals A validating the Miller and Modigliani proposition that dividends do not matter. A shortcut to a payout ratio is dividend yield times the P-E ratio or $(D/P)(P/E) = D/E = (1-b)$. A valuation using the price-earnings ratio follows from $P_0 = E_1(1-b)/(k-br)$ where $P-E = (1-b)/(k-br)$ and when multiplied by the expected earnings E_1 provides a stock valuation. In equilibrium when k equals r , the P-E ratio equals $1/k$.

Conclusion

While obvious once the mathematics are examined, I repeatedly ask my students what is the holding period return to bonds and stocks and rarely get a correct response. Thus I'm repeatedly reminded that this exercise is well worth the review.

References

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