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Intra-Industry Debt and Tacit Collusion

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Abstract

This paper shows that a firm competes less vigorously when it holds debt issued by another, competing firm. Reciprocal holding of debt by two firms may signal a credible commitment to collusion between the firms. This result is robust to a dynamic game setting, where reciprocal holding of debt is shown to reduce the firms' incentives to deviate from collusion. The message conveyed by these results is that intra-industry debt should raise concerns about tacit collusion. The results of this study are particularly relevant in the banking sector, where holding reciprocal debt is the norm, rather than the exception

Keywords: intra-industry debt; interbank lending; microeconomics of banking

JEL Classification Codes: G1, L13, L41

Introduction

This study is concerned with the effect of holdings of reciprocal debt between firms on the ability of the firms to collude in a product market. Lending and borrowing between firms that are otherwise competitors in their common product market may originate either in routine day to day trade in intermediate or final products between the firms, or in financial transactions. There may be various reasons for trading with a competitor.

For example, competitors may specialize in the production of some intermediate input, then trading that input between each other; they may save costs of transportation by trading in the final product, when their retail networks cover large geographical areas, as in the case of oil companies that are at the same time producers and distributors of gasoline.

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A particularly interesting case where extensive inter-firm lending and borrowing is the norm rather than the exception is banking. Banks hold net debt and credit positions with respect to other banks both in the short and long term. The literature on bank contagion is concerned with the destabilizing effect of extensive interbank lending. A recent attempt of estimating mutual credit positions in German banking is Upper and Worms (2002). For a sample of N banks, the authors estimate an $N \times N$ matrix X, where an entry x_{ij} is the lending position of bank i with respect to bank j. Furfine (1999) investigates bank contagion risks in the US based on a unique data set of bilateral bank exposures for US banks. Here is how the author describes his method.

"For example, suppose that Bank A lends Bank B $100 in the funds market. If Bank B fails, the simulations will assume that Bank A will lose either $40 or $5. If Bank A's capital level is 100, then Bank A will not fail, regardless of the assumed loss rate. If Bank A has only 15 in capital, Bank A will fail in one of the two calculations."

Though banking gives perhaps the most compelling justification for studying reciprocal debt holding, our focus is not on this particular sector. We believe credit arises in many other bilateral transactions between competing firms. Therefore, we will call reciprocal debt by the generic name of *trade credit*. Trade credit is a very short-term debt that arises between two firms engaged in market exchange of goods and services. The nature of this lending is informal, with the terms being subject to ex-post adjustment (Crawford, 1992).

The issue of how financial structure affects firm's behavior in the product market is not new. Brander and Lewis (1986) have formalized what they called the 'limited liability effect' of an oligopolistic industry's leverage. The authors argue that firms with higher level of debt tend to be more aggressive when competing in the product market.

The intuition is that a higher debt increases the likelihood that the firm fails. Under the threat of insolvency, the owners of the firm rationally take on more risk, because they cannot lose more than the value of the debt.

No work has been done so far on the case of competing firms' holding debt to each other. A segment of the literature that is closer to this idea is the one on 'partial ownership' (see for instance Reitman, 1994), where firms buy shares of each other, forming a 'partial ownership' arrangement.

Not surprisingly, such an arrangement proves to be competition hindering. The issue of inter-firm debt is fundamentally different from cross ownership. While the latter implies each firm caring about the other firm's profit, the former does this only to the extent that the rival firm may fail. Thus, each firm's incentive to participate in an implicit cartel is apparently lower in the case of inter-firm debt than when cross-ownership is present. Debt may also be regarded as a collusion-facilitating device that is subtler than cross ownership, because antitrust policy is more concerned with the equity structure of competing firms than with their debt structure. While equity structure is public information, trade credit is not necessarily publicly revealed.

A Static Model of a Duopoly with Mutual Debts

Let us consider two firms that compete in quantities in the product market. Denote their strategic variables (quantities) by x_i (i=1,2). The firms have debts D_i to each other, which can be any kind of trade credit. Without loss of generality, we assume that V^i, the value of Firm *i*, does not include other assets except for the operating profit of the firm, Π^i, plus outstanding credit or debt to the rival firm. Given the debt levels, the firms compete in the product market.

The outcome of the competition stage of the game depends on the strategies of the firms and on two unspecified random variables, one for each firm, which are two states of the world. There is a probability, p_i, that Firm *i* fails (becomes insolvent). Probabilities p_i are common knowledge, and they depend not only on the state of the world associated with Firm *i*, but also on debt D_i (see Appendix for a justification of this assumption). This assumption should be interpreted as follows.

Firm *i*'s operating profit is not random, as both demand and cost parameters are known. The source of uncertainty is, instead, overdue debt that a firm may owe to other creditors, or receivables that the firm does not receive back in time for meeting its operating costs or other debt obligations. Whether the creditors file for bankruptcy against the firm is uncertain. The creditors 'run' the firm, that is, initiate bankruptcy, with probability p_i.

When the reciprocal debts of the two firms are trade payables and receivables that appear in a normal course of business, it is unlikely that one of the firms would become insolvent because of this debt. Therefore, ‘profit’ in the expression of the value of the firm is an amount that is net of the debts to some third parties. Here is value of Firm *i*:

$$V^i(x_1,x_2)=\begin{cases}0, & if\ \Pi^i(x_1,x_2)\le D_i\\ \Pi^i(x_1,x_2)-D_i+p_j\Pi^j(x_1,x_2)+(1-p_j)D_j\,, & if\ \Pi^i(x_1,x_2)>D_i\end{cases}$$

The expected value of firm *i*is:

$$V^i(x_1,x_2)=(1-p_i)\left[\Pi^i(x_1,x_2)-D_i+p_j\Pi^j(x_1,x_2)+(1-p_j)D_j\right] \qquad (1)$$

In the unfavorable state of the world, which occurs with probability p_i, Firm *i*’s profit goes to Firm *j*, the creditor. The implicit assumption is that in the low state of the world Firm *i*’sprofit is less than its debt to Firm *j*. If Firm *i* does not fail, then it pays its debt D_i to Firm *j* and retains the remaining profit. The third term in (1) is the net profit that Firm *j* makes in its low state of the world (after creditors, other than Firm *i* have claimed their loans). Firm *j*’s profit in this case goes to Firm *i* on the account of Firm *j*’s debt. The last term is the receivable from Firm *j*, which is paid to Firm *i* only if Firm *j* does not become insolvent.

The firms maximize their profits by choosing quantities x_i and x_j, respectively. The first order conditions for maximizing the values of the firms (1) are:

$$V_i^i\equiv(1-p_i)\left(\Pi_i^i+p_j\Pi_i^j\right)=0\,,\quad i,j=(1,2),\ \ i\neq j \qquad (2)$$

Variables x_i have been omitted in equation (2), where the subscripts of the profit functions denote partial derivatives. We are interested in the effect of debts on the equilibrium output of the firms. Before looking at the equilibrium point, one may note that, if Firm *j* would have no debt to Firm *i*, there were no direct externalities between the two firms, and Firm *i* would be indifferent whether Firm *j* is insolvent. (It is assumed that ‘bankruptcy’ does not mean exit from the market, but change of owners. Thus, the surviving firm does not remain a monopoly if the rival becomes insolvent.) Without the debt, the second term in (2) would not exist.

The effect of changing p_j on the equilibrium values of the strategic variables can be determined by differentiating Equations (2) with respect to x_i, x_j, and p_i and calculating the following derivatives (see Appendix for details).

$$\frac{dx_j^*}{dp_i} = \frac{-(1-p_j)V_{ii}^i \Pi_j^i}{\Delta}, \qquad \frac{dx_i^*}{dp_i} = \frac{(1-p_j)V_{ij}^i \Pi_j^i}{\Delta}, \qquad (3)$$

$$\Delta = V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j .$$

In equations (3), the stars indicate the Nash equilibrium values of the strategic variables. For assessing the signs of the derivatives (3), one should note that the second order condition for maximizing the value functions (1) require V_{ii}^i and V_{jj}^j to be negative. Assume the reaction functions are downward sloping, which implies $\Delta > 0$. Consequently, relationship $|V_{ii}^i| > |V_{ij}^i|$ holds, where $|V_{ii}^i|$ stands for the absolute value of V_{ii}^i.

Proposition 1.A firm behaves less aggressively when its credit to the competing firm is higher.

<u>Proof:</u> The way Firm *j*'s behavior is influenced by Firm *i*'s debt is related to the sign of the derivative $\frac{dx_j^*}{dp_i}$, which is given by expressions (3).

This is so because p_i is increasing in firm *i*'sdebt, while the equilibrium solution does not depend explicitly on D_i, as can be noticed in the first order conditions.) In a quantity game, Π_j^i is negative. Taking into account the sign of the derivative V_{ii}^i, one can easily observe that $V_{ii}^i \Pi_j^i > 0$. Since $\Delta > 0$ by assumption, it turns out that $\frac{dx_j^*}{dp_i} < 0$, which shows that the higher the debt of Firm *i* to Firm *j* (measured indirectly by p_i), the lower Firm *j*'s equilibrium output. Under the assumption $\Pi_j^i < 0$, from the first order conditions (2) Π_j^j must be positive.

Therefore, a decrease in x_j can be associated with a less aggressive behavior of the firm.

Proposition 2.A firm behaves more aggressively when its debt to the competing firm is higher.

Proof: Following the same reasoning as in Proposition 1 and examining the sign of the derivative $\frac{dx_i^*}{dp_i}$ in equation (3), it can be observed that this derivative is positive.

Proposition 3.The industry is less competitive when reciprocal debts are higher.

Proof: The following derivative can be calculated using once more expressions (3).

$$\frac{d\left(x_i^* + x_j^*\right)}{dp_i} = \frac{\left(1 - p_j\right)}{\Delta}\Pi_j^{\prime i}\left(V_{ij}^i - V_{ii}^i\right),$$

which is negative under the assumptions discussed above.

Proposition 2 partially confirms Brander and Lewis's (1986) 'limited liability' effect, according to which a firm would behave more aggressively when it has debts. Proposition 3 reveals a limitation of the Brander and Lewis effect. According to Proposition 3, competition may be diminished when debts are reciprocal.

A last thing to check is under what conditions a solution involving positive mutual debts exists. To do that, we introduce an initial stage of the game, when the firms simultaneously choose their debts. To find the Nash equilibrium in debts, one needs to calculate the first-order conditions of the value-of-the-firm function (1) with respect to D_i and D_j, given the equilibrium quantities $x_i^*\left(D_i, D_j\right)$. After taking these derivatives and applying the envelope theorem, one can find that the equilibrium debts are positive when the following is true (in the symmetric case).

$$(1-p_i)\left(1-V_i^{\,j}\,\frac{dx_j^*}{dD_i}\right)+p_i\left(\pi^i+p_j\pi^j\right)<0\,.$$

This expression is still a function of the debts, but the signs of the terms involved are known. Thus, the second bracket can be negative, since both $V_i^{\,j}$ and $\frac{dx_j^*}{dD_i}$ are negative. This proves that a solution with positive debts exists. When the condition given by the equation above is not met, the two firms will choose zero debts.

Tacit Collusion in an Infinitely-Repeated Game

The goal of this section is to look at the behavior of the firms when they have mutual debts and they repeatedly interact in the market. Spagnolo (2000) suggests that leverage, as debt to third party creditors may have collusive consequences when interaction between stakeholders and managers are accounted for. Maksimovic (1988) finds a collusive effect of leverage in oligopoly. Since the focus of this paper is on reciprocal debt, the effects of outside credit are incorporated in probabilities p_i,the two 'state of the world' parameters.

The equilibrium concept used here is that of subgame perfect equilibrium (Abreu 1986, 1988). Dilip Abreu has shown that a maximum degree of collusion can be enforced in a repeated oligopoly game through a simple punishment strategy (penal code), and that strategies that are more complex cannot do better that the simple ones. Such a simple penal code is a subgame perfect equilibrium. It requires a 'stick and carrot' strategy, with a one-period severe punishment followed by return to the most cooperative stage game.

For a stick-and-carrot strategy to be credible, the one-period punishment payoff to the deviating firm should be such that, when added to the present value of the subsequent (infinite) stream of payoffs, the sum should amount to zero. A more severe punishment cannot be enforced since the punished firm can always earn zero discounted flow of payoffs by producing nothing forever. Such penal codes require that the punished firm genuinely participate in its own punishment. If it fails to do so, a new punishment period follows.

For modeling a repeated interaction between firms with reciprocal debts, let us assume that 'debt' is a fixed amount that each firm is to pay to the other firm, every period. Under this assumption, the payoffs to the firms are the value functions (1). Another important assumption is that the value functions meet the conditions for the existence of an optimal penal code.

The symbol $\hat{V}^{i}$ denotes the one-period value of Firm *i* when both firms produce the cooperative quantities. $\hat{V}^{i*}$ is the best-response (deviation) payoff of Firm *i*, when the rival keeps producing the cooperative output. $\overline{V}^{i}$ is the payoff to the punished firm at the end of the punishment ('stick') period. Combinations of these symbols will have suggestive meanings. In addition, ${}^{*}V$ is the payoff to a firm when the competitor deviates.

For any given discount rate, δ, an optimal simple penal code is an action undertaken by Firm *j* such that the following two incentive compatibility constraints hold for Firm *i*. Firm *j*'s action results in a one-period punishment payoff, $\overline{V}^{i}$, to Firm *i* (superscripts *i* are omitted).

$$\hat{V}^{*} - \hat{V} \leq \delta\left(\hat{V} - \overline{V}\right) \tag{4}$$

$$\overline{V}^{*} - \overline{V} \leq \delta\left(\hat{V} - \overline{V}\right) \tag{5}$$

Condition (4) reads that the gain from deviating one period during the cooperative phase should be less than the discounted loss from being punished the next period. A similar condition, (5), holds for the punishment phase, where $\hat{V}$ is the full-collusion payoff. A pair $\left(\underline{\delta}, \overline{V}\right)$ for which conditions (4) and (5) hold with equality, if such a pair exists, represents respectively the minimum discount rate, and the punishment payoff that can sustain full collusion.

Evaluating the effect of reciprocal debts on collusion is equivalent to studying how the minimal discount rate, $\underline{\delta}$, changes when *D*, the amount of debt, changes. In this context, the 'stick' payoff, $\overline{V}$, is not interesting, and it will be eliminated when solving equations (4) and (5).

Before finding the solution, $\underline{\delta}$, it is important to remember that an optimal punishment strategy requires that the discounted infinite stream of payoffs to the punished firm must be zero. This requirement is formalized in Equation (6):

$$\overline{V} + \frac{\delta}{1-\delta}\hat{V} = 0 \qquad (6)$$

Equation (6) can be transformed as follows:

$$\delta\left(\hat{V} - \overline{V}\right) = -\overline{V}.$$

This expression, together with (5) as equality, yields $\overline{V}^* = 0$. Solving equations (4) and (5) for $\underline{\delta}$, the breakeven value of the discount rate, yields:

$$\underline{\delta} = \frac{\hat{V}^* - \hat{V}}{\hat{V}^*} \qquad (7)$$

Let us recall that collusion is not sustainable for any rate lower than $\underline{\delta}$. Thus, any action causing an increase in $\underline{\delta}$ is pro-competitive, since it reduces the range where collusion is sustainable. On the contrary, lowering $\underline{\delta}$ is anti-competitive. Plugging the value of the firm, as given by Equation (1) into (7), the expression for $\underline{\delta}$ can be transformed as follows.

$$\underline{\delta}^i = \frac{\left(\hat{\Pi}^{i*} - \hat{\Pi}^i\right) - p_j\left(\hat{\Pi}^j - {}^*\hat{\Pi}^j\right)}{\hat{\Pi}^{i*} - D_i + p_j{}^*\hat{\Pi}^j + \left(1 - p_j\right)D_j} \qquad (8)$$

Proposition 4: Credit to a competing firm enhances creditor's incentive to collude.

Proof: Since cooperation is Pareto efficient, what Firm *i* gains from deviating, $\hat{\Pi}^{i*} - \hat{\Pi}^i$, must be equal to Firm *j*'s loss, $\hat{\Pi}^j -{}^* \hat{\Pi}^j$, where ${}^*\hat{\Pi}^j$ is the profit to Firm *j* when Firm *i* deviates in the collusion phase. Let us denote this quantity by *G*, and observe that *G>0*. This implies:

$$\hat{\Pi}^{i*} +^{*} \hat{\Pi}^{j} = \hat{\Pi}^{i} + \hat{\Pi}^{j} > 0 \qquad (9)$$

The effect of debt D_j on $\underline{\delta}^i$ is given by the sign of the derivative

$$\frac{\partial \underline{\delta}^i}{\partial D_j} = \frac{-G\left(\hat{\Pi}^{i*} - D_i + p_j{}^{*}\hat{\Pi}^{j} + (1-p_j)D_j\right) - (1-p_j)G\left({}^{*}\hat{\Pi}^{j} - D_j\right)}{\left(V^{i*}\right)^2}\frac{dp_j}{dD_j} - \frac{p_j(1-p_j)G}{\left(V^{i*}\right)^2}$$

$$= -G\left[\frac{\hat{\Pi}^{i*} - D_i +^{*} \hat{\Pi}^{j}}{\left(V^{i*}\right)^2}\frac{dp_j}{dD_j} + \frac{p_j(1-p_j)}{\left(V^{i*}\right)^2}\right]$$

$$= -G\left[\frac{\hat{\Pi}^{i} + \hat{\Pi}^{j} - D_i}{\left(V^{i*}\right)^2}\frac{dp_j}{dD_j} + \frac{p_j(1-p_j)}{\left(V^{i*}\right)^2}\right] . \qquad (10)$$

Assuming that Firm *i*'s collusive profit is higher than its one-period debt, one can easily notice that $\frac{\partial \underline{\delta}^i}{\partial D_j} < 0$, which is equivalent to a creditor's higher incentive to collude when credit goes up.

Proposition 5: Debt to a competing firm enhances debtor's incentive to collude.

Proof: Taking the derivative of expression (8) with respect to D_i, one can find:

$$\frac{\partial \underline{\delta}^i}{\partial D_i} = -\frac{(1-p_j)G}{\left(V^{i*}\right)^2} < 0 , \qquad (11)$$

which implies that the higher the debt to the competitor, the lower the breakeven discount rate, hence the higher the range over which collusion is sustainable.

Equation (8) provides an interesting insight. It shows that the only way in which own debt influences a firm's incentive to tacitly collude is by diminishing its profit from deviating, while the influence of the competitor's debt is more complex.

Proposition 6: Intra-industry credit facilitates tacit collusion.

Proof: According to propositions 4 and 5, the ranges of the discount rates are increased simultaneously for both firms when either debt or credit is present. This indicates that the industry as a whole is less competitive when firms maintain reciprocal debts.

Bankruptcy Costs

The question of how the previous results are influenced by the existence of bankruptcy costs makes the object of this section. For answering that question, derivatives (3), (10), and (11) will be recalculated.

Expression (1) of the value of the firm is modified to reflect costs of bankruptcy that are proportional to the gap between the 'assets,' which in this model are represented by the one-period profit, and the debt of the firm. Although in the finance literature bankruptcy costs are usually considered constant, a proportionality assumption appears more natural, as Brander and Lewis (1988) also noticed. Coefficient β stands for the fraction of the net debt that is spent in the process of bankruptcy. With this notation, the value-of-the firm function becomes:

$$V^i(x_1, x_2) = (1 - p_i)\left\{\Pi^i - D_i + p_j\left[\Pi^j - \beta\left(D_j - \Pi^j\right)\right] + \left(1 - p_j\right)D_j\right\} \tag{12}$$

Here are the derivatives of this function:

$$V_i^i \equiv (1 - p_i)\left[\Pi_i^i + p_j(1 + \beta)\Pi_i^j\right] = 0 \tag{13}$$

$$V_{ij}^i = (1 - p_i)\left[\Pi_{ij}^i + p_j(1 + \beta)\Pi_{ij}^j\right]$$

$$V_{ii}^i = (1 - p_i)\left[\Pi_{ii}^i + p_j(1 + \beta)\Pi_{ii}^j\right]$$

When all the derivatives of the profit functions that appear in the last expressions are negative, it can be easily noticed that derivatives (3) maintain their signs. Therefore, the results established in Propositions 1 to 3 hold when costs of bankruptcy are accounted for.

The next analysis focuses on the effect of costs of bankruptcy on the incentive to collude. Expression (12) can be used to calculate the breakeven interest rate (7). By straightforward calculations the following is obtained:

$$\underline{\delta}^i = \frac{G\left[1 - p_j(1+\beta)\right]}{\hat{\Pi}^{i*} - D_i + p_j\left[{}^*\hat{\Pi}^j - \beta\left(D_j - {}^*\hat{\Pi}^j\right)\right] + (1-p_j)D_j} \qquad (14)$$

Proposition 7: Costs of bankruptcy facilitate tacit collusion.

Proof: Taking derivative in (14) with respect to β, the following relationship can be determined.

$$Sign\left[\frac{\partial \underline{\delta}}{\partial \beta}\right] = Sign\left[-\hat{\Pi}^{i*} + D_i - {}^*\hat{\Pi}^j\right] \qquad (15)$$

Function V^i, as well as the interaction between firms makes sense only as long as Firm *i* receives a non-negative payment from Firm *j* in case of Firm *j*'s bankruptcy, that is:

${}^*\hat{\Pi}^j - \beta\left(D_j - {}^*\hat{\Pi}^j\right) \geq 0$, or $(1-\beta)\left({}^*\hat{\Pi}^j\right) - \beta D_j \geq 0$, which implies ${}^*\hat{\Pi}^j > 0$. With this observation, it is easy to see that the derivative in (15) is negative.

An intuition of Proposition 7 can be formulated by looking at expression (14). Since the bankruptcy costs are paid by the creditors, these costs act like increasing the debt, hence a higher probability of bankruptcy. The new probability of bankruptcy when bankruptcy costs are present is $p_j(1+\beta)$.

Summary and Conclusions

This paper is the first to investigate how interaction between firms in oligopoly is affected by the existence of reciprocal debts. The framework is a model of a Cournot duopoly, where firms may become insolvent because of outstanding debt to third creditors. Any event that prevents a firm to finance its due debt can trigger the firm's bankruptcy.

In a static setting, it has been proved that intra-industry credit lowers creditor's output and raises debtor's output, the latter result being consistent with the Brander and Lewis (1986) "limited liability" effect. The industry's total output is diminished by reciprocal credit, which is an anti-competitive effect.

When the firms interact repeatedly, intra-industry credit unambiguously enlarges the range of discount rates over which both firms have an incentive to collude. In other words, the anti-competitive effect that has been detected in a static setting is maintained when interaction is repetitive. Finally, it was proved that adding bankruptcy costs does not change these results. On the contrary, bankruptcy costs re-enforce the effects of mutual credit by increasing the probability of bankruptcy.

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Appendix

The Probability of Bankruptcy

Suppose Firm *i*has, besides its debt to Firm *j*, overdue debts to third parties amounting to K_i. Out of the firm's total profit, $\overline{\pi}^i$, an amount zK^i is paid out to third creditors, where $z \geq 0$ is a random variable with a cumulative distribution $F(z)$. Denote π^i the part of Firm *i*'s profit that remains after the debt K_i is paid, $\pi^i = \overline{\pi}^i - zK^i$. The probability that Firm *i*'s remaining profit falls short of the debt to Firm *j* is

$$p_i = P\left(\pi^i < D_i\right)$$
$$= P\left(\overline{\pi}^i - zK^i < D_i\right)$$
$$= P\left(z > \frac{\overline{\pi}^i - D^i}{K^i}\right)$$
$$= 1 - F\left(\frac{\overline{\pi}^i - D^i}{K^i}\right)$$

Using this expression for p_i, it is easy to see that $\frac{\partial p_1}{\partial D_1} > 0$.

Derivation of Equations (3)

The first-order conditions for maximizing the value of the firms V^i is $V_i^i = 0$, where *i=1, 2*. By differentiating these two equations with respect to x_i, x_j, and p_i the following equations are obtained:

$$\begin{cases} V_{ii}^i dx_i + V_{ij}^i dx_j + V_{ip_i}^i dp_i = 0 \\ V_{ji}^j dx_i + V_{jj}^j dx_j + V_{jp_i}^j dp_i = 0 \end{cases} \quad \text{(A1)}$$

Taking derivatives of V_i^i with respect to p_i, one can find the following cross-derivatives:

$$V_{ip_i}^i = -\left(\Pi_i^i + p_j \Pi_i^j\right) = -\frac{1}{1-p_i} V_i^i = 0, \text{ and } V_{jp_i}^j = \left(1-p_j\right)\Pi_j^i \quad \text{(A2)}$$

In Equations (A1) one can divide through by dp_i. For applying Cramer's rule of solving linear simultaneous equations, the following determinants are needed.

$$\Delta = \begin{vmatrix} V_{ii}^i & V_{ij}^i \\ V_{ji}^j & V_{jj}^j \end{vmatrix} = V_{ii}^i V_{jj}^j - V_{ji}^j V_{ij}^i,$$

$$\Delta_i = \begin{vmatrix} -V^i_{ip_i} & V^i_{ij} \\ -V^j_{jp_i} & V^j_{jj} \end{vmatrix} = -V^i_{ip_i} V^j_{jj} + V^j_{jp_i} V^i_{ij} = (1 - p_j)\Pi^j_i V^i_{ij},$$

$$\Delta_j = \begin{vmatrix} V^i_{ii} & -V^i_{ip_i} \\ V^j_{ji} & -V^j_{jp_i} \end{vmatrix} = -V^i_{ii} V^j_{jp_i} + V^j_{ji} V^i_{ip_i} = -(1 - p_j)\Pi^j_i V^i_{ii}.$$

Finally, expressions (2) can be found by applying Cramer's rule:

$$\frac{dx_i^*}{dp_i} = \frac{\Delta_i}{\Delta}, \qquad \frac{dx_j^*}{dp_i} = \frac{\Delta_j}{\Delta}$$